



# High School Precalculus: Trigonometry

## SY 2022/2023

# High School Precalculus: Trigonometry

## Units of Study

<b>Unit 1:</b>	Right Trig and Angle Measures	🕒	15 days	1st semester
<b>Unit 2:</b>	The Unit Circle and Graphs of Trig Functions	🕒	15 days	1st semester
<b>Unit 3:</b>	Inverse Trig and Trig Identities	🕒	15 days	1st semester
<b>Unit 4:</b>	Other Trig Identities and Trig Equations	🕒	15 days	1st semester
<b>Unit 5:</b>	Applications of Trig Functions	🕒	15 days	1st semester
<b>Unit 6:</b>	Polar Coordinates and Vectors	🕒	15 days	1st semester

## Appendices

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**Appendix A:** [Proficiency Scale Template](#)

**Appendix B:** [Curriculum Refinement Form](#)

**Appendix C:** [North Gibson Priority Standards Vertical Articulation Document](#)

# High School Precalculus: Trigonometry Priority Standards

<b>Priority Standards</b>	<b>TR.ID.2</b>	Verify trigonometric identities and simplify expressions using trigonometric identities.
	<b>TR.PF.1</b>	Graph trigonometric functions with and without technology. Use the graphs to model and analyze periodic phenomena, stating amplitude, period, frequency, phase shift, and midline (vertical shift).
	<b>TR.PF.5</b>	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
	<b>TR.T.2</b>	Solve real-world problems with and without technology that can be modeled using right triangles, including problems that can be modeled using trigonometric ratios. Interpret the solutions and determine whether the solutions are reasonable.
	<b>TR.T.5</b>	Understand and apply the Laws of Sines and Cosines to solve real-world and other mathematical problems involving right and non-right triangles.
	<b>TR.UC.3</b>	Use special triangles to determine the values of sine, cosine, and tangent for $\pi/3$ , $\pi/4$ , and $\pi/6$ . Apply special right triangles to the unit circle and use them to express the values of sine, cosine, and tangent for $x$ , $\pi \pm x$ , and $2\pi \pm x$ in terms of their values for $x$ , where $x$ is any real number.
	<b>TR.V.7</b>	Solve problems involving velocity and other quantities that can be represented by vectors.

# Standards Breakdown

: Priority Standards

: Supporting Standards

: Additional Standards

		UNITS					
		1	2	3	4	5	6
Identities	1			•			
	2			★	★		
	3				•		
	4				•		
Polar Coordinates	1						★
	2						•
	3						•
	4						•
Periodic Functions	1		★				
	2					•	
	3		•				
	4			•			
	5				•		
Triangles	1	•					
	2	★					
	3	•					
	4					—	
	5					★	
	6					•	
Unit Circle	1	•					
	2	•					
	3		★				
Vectors	1						•
	2						•
	3						•
	4						•
	5						•
	6						•
	7						★

<b>General Description of the Unit</b> This unit builds substantially on students' knowledge of the trigonometric ratios. Radians are introduced and the definition of trigonometric functions is expanded to angles greater than 90°. Real-world and mathematical problems involving right triangles are also solved.		
<b>Priority Standards</b> <ul style="list-style-type: none"> <li>• <b>TR.T.2:</b> Solve real-world problems with and without technology that can be modeled using right triangles, including problems that can be modeled using trigonometric ratios. Interpret the solutions and determine whether the solutions are reasonable.</li> </ul>	<b>Supporting Standards</b> <ul style="list-style-type: none"> <li>• <b>TR.T.1:</b> Define and use the trigonometric ratios (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles and the coordinates on the unit circle.</li> <li>• <b>TR.T.3:</b> Explain and use the relationship between the sine and cosine of complementary angles.</li> <li>• <b>TR.UC.1:</b> Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</li> <li>• <b>TR.UC.2:</b> Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</li> </ul>	
<b>Enduring Understandings</b> <ul style="list-style-type: none"> <li>• Many real-world situations can be modeled with right triangles and solved using trigonometric ratios. Any solutions must be checked against the context to ensure reasonableness.</li> <li>• Radians are another way to measure angles, where angles are measured by the distance traveled along the circle relative to the radius.</li> <li>• Radians and degrees are related, and we can translate between the two measurements.</li> <li>• We can evaluate the value of trigonometric functions for all angle measures.</li> </ul>	<b>Essential Questions</b> <ul style="list-style-type: none"> <li>• How might a carpenter use the trigonometric ratios?</li> <li>• Why does <math>\sin(40^\circ) = \cos(50^\circ)</math>?</li> <li>• What is the benefit of using radians instead of degrees to measure angles?</li> </ul>	
<b>Key Concepts</b> <ul style="list-style-type: none"> <li>• I can solve real-world problems that can be modeled using right triangles with and without technology. (TR.T.2)</li> <li>• I can use trigonometric ratios to solve real-world problems that can be modeled with right triangles with and without technology. (TR.T.2)</li> <li>• I can interpret the solutions to determine whether they are reasonable within the context of the problem. (TR.T.2)</li> </ul>	<b>Related Concepts</b> <ul style="list-style-type: none"> <li>• I can define all trigonometric ratios and their inverses in terms of angles of right triangles and the coordinates on the unit circle. (TR.T.1)</li> <li>• I can use the unit circle and the six trigonometric ratios to solve problems. (TR.T.1)</li> <li>• I can investigate the relationship between the sine and cosine of complementary angles. (TR.T.3)</li> <li>• I can use the idea that sine and cosine are cofunctions to solve problems. (TR.T.3)</li> <li>• I can identify angles subtended by arcs. (TR.UC.1)</li> <li>• I can define the angle created when the length of an arc of a circle is the same as the length of the circle's radius as a radian. (TR.UC.1)</li> <li>• I can define the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (TR.UC.1)</li> </ul>	<b>Vocabulary</b> <ul style="list-style-type: none"> <li>• Arc</li> <li>• Central angle</li> <li>• Cofunction</li> <li>• Complementary angles</li> <li>• Coordinate plane</li> <li>• Cosecant</li> <li>• Cosine</li> <li>• Cotangent</li> <li>• Coterminal angle</li> <li>• Radian measure</li> <li>• Real numbers</li> <li>• Reference angle</li> <li>• Right triangle</li> <li>• Secant</li> <li>• Sine</li> <li>• Tangent</li> <li>• Theta</li> <li>• Trigonometric functions</li> <li>• Trigonometric ratios</li> <li>• Unit circle</li> </ul>

- I can recognize that the coordinates of any point on the unit circle may be defined as  $(\cos \theta, \sin \theta)$ . (TR.UC.2)
- I can identify that  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ . (TR.UC.2)
- I can find the measure of the angle and state the trigonometric ratio for that angle when given a point on the coordinate plane. (TR.UC.2)
- I can graph trigonometric functions on the unit circle. (TR.UC.2)

**Mathematical Processes**

- PS.1 Make sense of problems and persevere in solving them.
- PS.6 Attend to precision.

**Resources**

**Proficiency Scales**

- [TR.T.2](#)

**Digital**

- [IDOE Examples/Tasks TR.T.2](#)
- [IDOE Examples/Tasks TR.T.1](#)
- [IDOE Examples/Tasks TR.T.3](#)
- [IDOE Examples/Tasks TR.UC.1](#)
- [IDOE Examples/Tasks TR.UC.2](#)

**Manipulatives**

- [Geogebra Geometry](#)
- [Graphing Calculator](#)
- [Unit Circle](#)

**School Resources**

**Textbook**

Textbook: Indiana Reveal by McGraw-Hill

- Ch 5: Trigonometric Functions
- 5.1 Angles and Their Measure
- 5.2 Right Triangle Trigonometry
- 5.3 Trigonometric Functions of Any Angle
- 7.1 Applications of Right Triangles

**Formative Assessments**

General Description of the Unit		
<p>Now the unit circle is developed and explored. Additionally, the graphs of the trigonometric functions and inverse trigonometric functions are produced, along with transformations of these functions. These functions are used for modeling real-world situations.</p>		
<p><b>Priority Standards</b></p> <ul style="list-style-type: none"> <li>• <b>TR.PF.1:</b> Graph trigonometric functions with and without technology. Use the graphs to model and analyze periodic phenomena, stating amplitude, period, frequency, phase shift, and midline (vertical shift).</li> <li>• <b>TR.UC.3:</b> Use special triangles to determine the values of sine, cosine, and tangent for <math>\pi/3</math>, <math>\pi/4</math>, and <math>\pi/6</math>. Apply special right triangles to the unit circle and use them to express the values of sine, cosine, and tangent for <math>x</math>, <math>\pi \pm x</math>, and <math>2\pi \pm x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</li> </ul>	<p><b>Supporting Standards</b></p> <ul style="list-style-type: none"> <li>• <b>TR.PF.3:</b> Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</li> </ul>	
<p><b>Enduring Understandings</b></p> <ul style="list-style-type: none"> <li>• Trigonometric functions, especially sine and cosine, are often good models for periodic phenomena.</li> <li>• Trigonometric functions are periodic, with the same values repeating around the unit circle.</li> <li>• We can use special right triangles to evaluate key trigonometric values, including angles with multiple rotations around the unit circle.</li> </ul>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>• If given a graph of a trigonometric function, is it possible to find more than one equation to match the graph? Why or why not?</li> <li>• What are two angle measures that have the same cosine value? How do you know?</li> <li>• Does the transformation of a sine function always result in an even function? Periodic? Why or why not?</li> </ul>	
<p><b>Key Concepts</b></p> <ul style="list-style-type: none"> <li>• I can graph trigonometric functions with technology. (TR.PF.1)</li> <li>• I can graph trigonometric functions without technology. (TR.PF.1)</li> <li>• I can identify amplitude, frequency, period, and midline given a graph of a trigonometric function. (TR.PF.1)</li> <li>• I can use similarity to determine the side measures of <math>30^\circ</math>-<math>60^\circ</math>-<math>90^\circ</math> and <math>45^\circ</math>-<math>45^\circ</math>-<math>90^\circ</math> triangles. (TR.UC.3)</li> <li>• I can find the values of sine, cosine, and tangent in the special right triangle using degree and radian measures. (TR.UC.3)</li> <li>• I can understand and use reference angles on the unit circle. (TR.UC.3)</li> </ul>	<p><b>Related Concepts</b></p> <ul style="list-style-type: none"> <li>• I can define cosine and secant as being even functions and symmetric to the y-axis. (TR.PF.3)</li> <li>• I can define sine, cosecant, tangent, and cotangent as being odd and symmetric to the origin. (TR.PF.3)</li> <li>• I can compare values of trigonometric functions in quadrants I and IV to determine whether a function is even or odd. (TR.PF.3)</li> </ul>	<p><b>Vocabulary</b></p> <ul style="list-style-type: none"> <li>• Amplitude</li> <li>• Angle of rotation</li> <li>• Cosine</li> <li>• Coterminal</li> <li>• Domain</li> <li>• Even symmetry</li> <li>• Frequency</li> <li>• Initial side</li> <li>• Midline</li> <li>• Odd symmetry</li> <li>• Period</li> <li>• Periodic phenomena</li> <li>• Phase shift</li> <li>• Reference angle</li> <li>• Sine</li> <li>• Special right triangles</li> <li>• Tangent</li> <li>• Terminal side</li> <li>• Trigonometric functions</li> <li>• Unit circle</li> </ul>
<p><b>Mathematical Processes</b></p> <ul style="list-style-type: none"> <li>• PS.2 Reason abstractly and quantitatively.</li> <li>• PS.4 Model with mathematics.</li> </ul>		
Resources		
<p><b>Proficiency Scales</b></p> <ul style="list-style-type: none"> <li>• <a href="#">TR.PF.1</a></li> <li>• <a href="#">TR.UC.3 – Blank Template</a></li> </ul>	<p><b>Digital</b></p> <ul style="list-style-type: none"> <li>• <a href="#">IDOE Examples/Tasks TR.PF.1</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.UC.3</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.PF.3</a></li> </ul>	<p><b>Manipulatives</b></p> <ul style="list-style-type: none"> <li>• <a href="#">Coordinate Grid</a></li> <li>• <a href="#">Graphing Calculator</a></li> <li>• <a href="#">Unit Circle</a></li> </ul>

## School Resources

### Textbook

Ch 5: Trigonometric Functions

5.4 Trigonometric Functions Defined on the Unit Circle

5.5 Graphs of Sine and Cosine Functions

5.6 Graphs of Other Trigonometric Functions

### Formative Assessments



## Unit 3: Inverse Trig and Trig Identities (15 days, 1st semester)

General Description of the Unit		
<p>This unit and the next unit explore a variety of trigonometric identities. Students should be able to prove key identities, in addition to using those identities to solve problems. Then students will verify new identities using algebra and will use those identities to simplify trigonometric expressions. Inverse trigonometric functions are also developed and utilized.</p>		
<p><b>Priority Standards</b></p> <ul style="list-style-type: none"> <li>• <b>TR.ID.2:</b> Verify trigonometric identities and simplify expressions using trigonometric identities.</li> </ul>	<p><b>Supporting Standards</b></p> <ul style="list-style-type: none"> <li>• <b>TR.ID.1:</b> Prove the Pythagorean identity <math>\sin^2(x) + \cos^2(x) = 1</math> and use it to find trigonometric ratios, given <math>\sin(x)</math>, <math>\cos(x)</math>, or <math>\tan(x)</math>, and the quadrant of the angle.</li> <li>• <b>TR.PF.4:</b> Construct the inverse trigonometric functions of sine, cosine, and tangent by restricting the domain.</li> </ul>	
<p><b>Enduring Understandings</b></p> <ul style="list-style-type: none"> <li>• There are multiple trigonometric identities; these can be used to put expressions and equations into an easier form to evaluate.</li> <li>• The Pythagorean identity gives us another method for evaluating trigonometric ratios. We must be careful to use the angle's quadrant to select the correct sign for each ratio.</li> <li>• Because sine, cosine, and tangent functions are not one-to-one, their domain must be restricted before finding their inverse.</li> </ul>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>• How do the additional trigonometric identities allow us to evaluate the trigonometric value of additional angle measures without the use of a calculator?</li> <li>• If we know <math>\cos(x) = 4/5</math>, do we have enough information to find <math>\sin(x)</math>? Why or why not?</li> <li>• How can we find more than one domain restriction to make the cosine function invertible?</li> </ul>	
<p><b>Key Concepts</b></p> <ul style="list-style-type: none"> <li>• I can validate trigonometric identities. (TR.ID.2)</li> <li>• I can simplify expressions using trigonometric identities. (TR.ID.2)</li> <li>• I can use a variety of trigonometric identities to simplify expressions. (TR.ID.2)</li> </ul>	<p><b>Related Concepts</b></p> <ul style="list-style-type: none"> <li>• I can use the Pythagorean Theorem to prove the Pythagorean identities. (TR.ID.1)</li> <li>• I can use Pythagorean identities to find trigonometric ratios given sine, cosine, or tangent and the quadrant of the angle. (TR.ID.1)</li> <li>• I can restrict a domain in order to construct the inverse functions of sine, cosine, and tangent. (TR.PF.4)</li> </ul>	<p><b>Vocabulary</b></p> <ul style="list-style-type: none"> <li>• Cosine</li> <li>• Domain</li> <li>• Inverse trigonometric functions</li> <li>• Pythagorean Identity</li> <li>• Quadrant</li> <li>• Sine</li> <li>• Tangent</li> <li>• Trigonometric identities</li> <li>• Trigonometric ratios</li> </ul>
<p><b>Mathematical Processes</b></p> <ul style="list-style-type: none"> <li>• PS.3 Construct convincing arguments and critique the reasoning of others.</li> <li>• PS.8 Look for and express regularity in repeated reasoning.</li> </ul>		
Resources		
<p><b>Proficiency Scales</b></p> <ul style="list-style-type: none"> <li>• <a href="#">TR.ID.2</a></li> </ul>	<p><b>Digital</b></p> <ul style="list-style-type: none"> <li>• <a href="#">IDOE Examples/Tasks TR.ID.2</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.ID.1</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.PF.4</a></li> </ul>	<p><b>Manipulatives</b></p> <ul style="list-style-type: none"> <li>• <a href="#">Coordinate Grid</a></li> <li>• <a href="#">Graphing Calculator</a></li> </ul>

## School Resources

### Textbook

5.7 Inverse Trigonometric Functions  
6.1 Fundamental Trigonometric Identities

### Formative Assessments

## Unit 4: Other Trig Identities and Trig Equations (15 days, 1st semester)

General Description of the Unit			
<p>Now more trigonometric identities are developed and applied. Additionally, inverse trig functions are used to solve equations and real-world problems.</p>			
<p><b>Priority Standards</b></p> <ul style="list-style-type: none"> <li>• <b>TR.PF.5:</b> Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.</li> <li>• <b>TR.ID.2:</b> Verify trigonometric identities and simplify expressions using trigonometric identities.</li> </ul>	<p><b>Supporting Standards</b></p> <ul style="list-style-type: none"> <li>• <b>TR.ID.3:</b> Prove the addition and subtraction identities for sine, cosine, and tangent. Use the identities to solve problems.</li> <li>• <b>TR.ID.4:</b> Prove the double- and half-angle identities for sine, cosine, and tangent. Use the identities to solve problems.</li> </ul>		
<p><b>Enduring Understandings</b></p> <ul style="list-style-type: none"> <li>• Inverse trigonometric functions are often used to solve for unknown angle measures in a real life situation or problems involving force being applied to an inclined object. Any solutions must be checked against the context to ensure reasonableness.</li> <li>• The sum and difference identities give us additional methods for evaluating trigonometric ratios.</li> <li>• The double-angle and half-angle identities give us additional methods for evaluating trigonometric ratios.</li> </ul>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>• What types of real-world scenarios would the inverse trigonometric functions give us helpful information about?</li> <li>• What is an angle measure whose trigonometric value would be mostly easily found using a subtraction identity? Why?</li> <li>• What is an angle measure whose trigonometric value would be mostly easily found using a double-angle identity? Why?</li> </ul>		
<p><b>Key Concepts</b></p> <ul style="list-style-type: none"> <li>• I can use inverse functions to solve trigonometric equations that arise in modeling contexts. (TR.PF.5)</li> <li>• I can interpret the solution using technology and determine if there are additional solutions. (TR.PF.5)</li> <li>• I can interpret the solution in terms of the context of the model. (TR.PF.5)</li> <li>• I can validate trigonometric identities. (TR.ID.2)</li> <li>• I can simplify expressions using trigonometric identities. (TR.ID.2)</li> <li>• I can use a variety of trigonometric identities to simplify expressions. (TR.ID.2)</li> </ul>	<p><b>Related Concepts</b></p> <ul style="list-style-type: none"> <li>• I can prove the addition and subtraction identities for sine, cosine, and tangent. (TR.ID.3)</li> <li>• I can use the addition and subtraction identities for sine, cosine, and tangent to solve problems. (TR.ID.3)</li> <li>• I can prove the double-angle identity for sine, cosine, and tangent using the addition identities. (TR.ID.4)</li> <li>• I can prove the half-angle identity for sine, cosine, and tangent. (TR.ID.4)</li> <li>• I can use the double- and half-angle identities for sine, cosine, and tangent to solve problems. (TR.ID.4)</li> </ul>	<p><b>Vocabulary</b></p> <ul style="list-style-type: none"> <li>• Cosine</li> <li>• Displacement</li> <li>• Double-angle formula</li> <li>• Equilibrium</li> <li>• Frequency</li> <li>• Half-angle formula</li> <li>• Harmonic motion</li> <li>• Inverse function</li> <li>• Midline</li> <li>• Periodic</li> <li>• Sine</li> <li>• Sinusoidal</li> <li>• Tangent</li> <li>• Trigonometric equations</li> <li>• Trigonometric identities</li> <li>• Wave speed</li> </ul>	
<p><b>Mathematical Processes</b></p> <ul style="list-style-type: none"> <li>• PS.3 Construct convincing arguments and critique the reasoning of others.</li> <li>• PS.4 Model with mathematics.</li> </ul>			
Resources			
<p><b>Proficiency Scales</b></p> <ul style="list-style-type: none"> <li>• <a href="#">TR.ID.2</a></li> <li>• <a href="#">TR.PF.5 – Blank Template</a></li> </ul>	<p><b>Digital</b></p> <ul style="list-style-type: none"> <li>• <a href="#">IDOE Examples/Tasks TR.ID.2</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.PF.5</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.ID.3</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.ID.4</a></li> </ul>	<p><b>Manipulatives</b></p> <ul style="list-style-type: none"> <li>• <a href="#">Coordinate Grid</a></li> <li>• <a href="#">Graphing Calculator</a></li> </ul>	

## School Resources

### Textbook

6.2 Sum and Difference Formulas  
6.3 Double-Angle, Power-Reducing, and Half-Angle Formulas  
6.4 Product-to-Sum and Sum-to-Product Formulas  
6.5 Trigonometric Equations

### Formative Assessments

General Description of the Unit		
This unit moves beyond the theory and graphing of trigonometry to explore multiple modeling applications of trigonometry. Students will model with all triangles, not just right triangles, using the Laws of Sine and Cosine and a new area formula. They will also use a sinusoidal function as a model of a bivariate data.		
<b>Priority Standards</b> <ul style="list-style-type: none"> <li>• <b>TR.T.5:</b> Understand and apply the Laws of Sines and Cosines to solve real-world and other mathematical problems involving right and non-right triangles.</li> </ul>	<b>Supporting Standards</b> <ul style="list-style-type: none"> <li>• <b>TR.PF.2:</b> Model a data set with periodicity using a sinusoidal function and explain the parameters of the model.</li> <li>• <b>TR.T.6:</b> Derive the formula <math>A = \frac{1}{2} ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line. Use the formula to find areas of triangles.</li> </ul>	
		<b>Additional Standards</b> <ul style="list-style-type: none"> <li>• <b>TR.T.4:</b> Prove the Laws of Sines and Cosines.</li> </ul>
<b>Enduring Understandings</b> <ul style="list-style-type: none"> <li>• The Laws of Sines and Cosines give us another way to solve for the sides and angles of non-right triangles. In the Law of Sines, we need to look out for certain scenarios where there could be two solutions.</li> <li>• Sinusoidal functions are often good models for bivariate data that appears periodic.</li> <li>•</li> </ul>	<b>Essential Questions</b> <ul style="list-style-type: none"> <li>• What types of triangles are best solved using the Law of Sines? Be as specific as possible.</li> <li>• What is a data set that would likely be best modeled with a sinusoidal function?</li> <li>• When calculating the area, what types of triangles (if any) would you prefer to use the formula <math>A = \frac{1}{2} ab \sin(C)</math> instead of the traditional area formula? Why?</li> </ul>	
<b>Key Concepts</b> <ul style="list-style-type: none"> <li>• I can explain the Laws of Sines and Cosines. (TR.T.5)</li> <li>• I can determine when the use of the Laws of Sines and Cosines is necessary. (TR.T.5)</li> <li>• I can use the Laws of Sines and Cosines to solve real-world problems. (TR.T.5)</li> </ul>	<b>Related Concepts</b> <ul style="list-style-type: none"> <li>• I can model a data set with periodicity using a sinusoidal function. (TR.PF.2)</li> <li>• I can explain the parameters of a sinusoidal model. (TR.PF.2)</li> <li>• I can derive the area formula for a triangle using the sine ratio. (TR.T.6)</li> <li>• I can determine the area of a triangle given a variety of information. (TR.T.6)</li> <li>• I can prove the Laws of Sines and Cosines by drawing and using altitudes in both acute and obtuse triangles. (TR.T.4)</li> </ul>	<b>Vocabulary</b> <ul style="list-style-type: none"> <li>• Altitude</li> <li>• Auxiliary line</li> <li>• Complementary angles</li> <li>• Law of Cosine</li> <li>• Law of Sines</li> <li>• Parameter</li> <li>• Right triangle</li> <li>• Sinusoidal function</li> <li>• Triangle Sum Theorem</li> </ul>
<b>Mathematical Processes</b> <ul style="list-style-type: none"> <li>• PS.1 Make sense of problems and persevere in solving them.</li> <li>• PS.8 Look for and express regularity in repeated reasoning.</li> </ul>		
Resources		
<b>Proficiency Scales</b> <ul style="list-style-type: none"> <li>• <a href="#">TR.T.5</a></li> </ul>	<b>Digital</b> <ul style="list-style-type: none"> <li>• <a href="#">IDOE Examples/Tasks TR.T.5</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.PF.2</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.T.6</a></li> <li>• <a href="#">IDOE Examples/Tasks TR.T.4</a></li> </ul>	<b>Manipulatives</b> <ul style="list-style-type: none"> <li>• <a href="#">Coordinate Grid</a></li> <li>• <a href="#">Graphing Calculator</a></li> </ul>

## School Resources

### Textbook

Ch 7: Applications of Trigonometric Functions

7.2 The Law of Sines

7.3 The Law of Cosines

7.4 Harmonic Motion

### Formative Assessments

**General Description of the Unit**

**Polar Coordinate:** Students will extend their knowledge of complex numbers to polar form. They will also prove and use DeMoivre’s Theorem. Then students will work on the polar plane, including translating coordinates and equations between polar and rectangular form.

**Vectors:** Students will be introduced to vectors, perform multiple vector operations, and use vectors to model real-world situations. In the introduction to vectors, students will understand a vector as having magnitude and direction, along with exploring the components of a vector. When doing vector operations, students will understand and perform vector addition, subtraction, and multiplication. Finally, students will use vectors to model physical situations, such as velocity and force, and will solve these problems using the properties of vectors.

**Priority Standards**

- **TR.V.7:** Solve problems involving velocity and other quantities that can be represented by vectors.

**Supporting Standards**

- **TR.PC.1:** Understand and use complex numbers, including real and imaginary numbers, on the complex plane in rectangular and polar form, and explain why the rectangular and polar forms of a given complex number represent the same number.
- **TR.PC.2:** State, prove, and use DeMoivre’s Theorem.
- **TR.PC.3:** Define polar coordinates and relate polar coordinates to Cartesian coordinates.
- **TR.PC.4:** Translate equations from rectangular coordinates to polar coordinates and from polar coordinates to rectangular coordinates. Graph equations in the polar coordinate plane.
- **TR.V.1:** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ).
- **TR.V.2:** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **TR.V.3:** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **TR.V.4:** Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **TR.V.5:** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- **TR.V.6:** Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ).

<p><b>Enduring Understandings</b></p> <ul style="list-style-type: none"> <li>• DeMoivre's Theorem gives us a more direct method for simplifying powers of complex numbers than the binomial theorem.</li> <li>• Polar coordinates are another way of graphing points. They define a point as <math>(r, \theta)</math>, where <math>\theta</math> represents the direction and <math>r</math> is the distance from the origin.</li> <li>• Vectors have both direction and magnitude, which makes them useful for modeling many physical phenomena such as wind or force.</li> </ul>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>• Will the graphs of <math>y = \sin(x)</math> and <math>r = \sin(\theta)</math> look the same? Why or why not?</li> <li>• Why are vectors so commonly used in physics?</li> <li>• What do the components of a vector tell us about the vector?</li> <li>• What is a scenario that could be modeled by vector subtraction?</li> </ul>	
<p><b>Key Concepts</b></p> <ul style="list-style-type: none"> <li>• I can solve problems involving velocity. (TR.V.7)</li> <li>• I can solve problems involving quantities that can be represented by vectors. (TR.V.7)</li> </ul>	<p><b>Related Concepts</b></p> <ul style="list-style-type: none"> <li>• I can identify complex numbers. (TR.PC.1)</li> <li>• I can identify the real and imaginary parts of a complex number. (TR.PC.1)</li> <li>• I can graph complex numbers on the complex plane in rectangular form. (TR.PC.1)</li> <li>• I can graph complex numbers on the complex plane in polar form. (TR.PC.1)</li> <li>• I can translate between rectangular and polar forms of a complex number. (TR.PC.1)</li> <li>• I can justify why the rectangular and polar forms of a complex number represent the same number. (TR.PC.1)</li> <li>• I can state DeMoivre's Theorem. (TR.PC.2)</li> <li>• I can use induction to prove De Moivre's Theorem. (TR.PC.2)</li> <li>• I can use DeMoivre's Theorem. (TR.PC.2)</li> <li>• I can connect complex numbers and trigonometry using DeMoivre's Theorem. (TR.PC.2)</li> <li>• I can use DeMoivre's Theorem to find powers and roots of complex numbers. (TR.PC.2)</li> <li>• I can find the absolute value and argument of a complex number in order to apply De Moivre's Theorem. (TR.PC.2)</li> <li>• I can use the unit circle or trig functions to identify theta in order to use DeMoivre's Theorem. (TR.PC.2)</li> <li>• I can define polar coordinates simplistically as a point marked by how far away from the origin and what angle it is with the x axis. (TR.PC.3)</li> <li>• I can relate polar coordinates to Cartesian coordinates. (TR.PC.3)</li> <li>• I can translate equations from rectangular coordinates to polar</li> </ul>	<p><b>Vocabulary</b></p> <ul style="list-style-type: none"> <li>• Absolute value of a complex number</li> <li>• Argument of a complex number</li> <li>• Cartesian Coordinates</li> <li>• Complex number</li> <li>• Complex plane</li> <li>• DeMoivre's Theorem</li> <li>• Imaginary number</li> <li>• Magnitude</li> <li>• Polar coordinate plane</li> <li>• Polar coordinates</li> <li>• Polar form</li> <li>• Pythagorean Theorem</li> <li>• Real number</li> <li>• Rectangular coordinates</li> <li>• Rectangular form</li> <li>• Scalar multiple</li> <li>• Scalar multiplication</li> <li>• Trigonometry</li> <li>• Vector</li> <li>• Velocity</li> </ul>



- coordinates using the Pythagorean Theorem and trigonometric ratios. (TR.PC.4)
- I can convert from polar coordinates to rectangular coordinates using trigonometric ratios. (TR.PC.4)
- I can graph equations in the polar coordinate plane. (TR.PC.4)
- I can represent scalar multiplication graphically by scaling vectors. (TR.V.5)
- I can reverse vector direction to represent scalar multiplication graphically. (TR.V.5)
- I can perform scalar multiplication component-wise. (TR.V.5)
- I can compute the magnitude of a scalar multiple. (TR.V.6)
- I can compute the direction of a vector. (TR.V.6)

**Mathematical Processes**

- PS.5 Use tools appropriately.
- PS.7 Look for and make use of structure.

**Resources**

**Proficiency Scales**

- [TR.V.7](#)

**Digital**

- [IDOE Examples/Tasks TR.PC.1](#)
- [IDOE Examples/Tasks TR.V.7](#)
- [IDOE Examples/Tasks TR.PC.2](#)
- [IDOE Examples/Tasks TR.PC.3](#)
- [IDOE Examples/Tasks TR.PC.4](#)
- [IDOE Examples/Tasks TR.V.5](#)
- [IDOE Examples/Tasks TR.V.6](#)

**Manipulatives**

- [Coordinate Grid](#)
- [Geogebra](#)
- [Graphing Calculator](#)
- [Polar Coordinate Graph Paper](#)

**School Resources**

**Textbook**

Ch 8: Trigonometry Applied to Polar Coordinate Systems and Vectors  
 8.1 Polar Coordinates  
 8.2 Graphs of Polar Equations  
 8.3 Complex Numbers in the Polar Form  
 8.4 Vectors  
 8.5 Dot Product (SKIP - not an IAS)

**Formative Assessments**